Solution 18 :

1°/ a) θ étant un argument de ω, il reste à trouver $|\omega| = \rho$.

$\omega = \rho\cos\theta + i\rho\sin\theta$ or $\rho\sin\theta = tg\,\theta \Rightarrow \rho = \dfrac{1}{\cos\theta}$

$\omega = \left[\dfrac{1}{\cos\theta}, \theta\right] = \dfrac{1}{\cos\theta} e^{i\theta}$

$\omega = \dfrac{1}{\cos\theta}(\cos\theta + i\sin\theta)$

b) on a : $\omega = \dfrac{1}{\cos\theta}(\cos\theta + i\sin\theta) = 1 + i\,tg\,\theta$

Donc $\omega = 1 + i\,tg\,\theta$

2°/ $2z^2 - (1+i\sqrt{3})z - 1 + i\sqrt{3} = 0$

$\Delta = (1+i\sqrt{3})^2 - 8(-1+i\sqrt{3}) = -2 + 2i\sqrt{3} + 8 - 8i\sqrt{3}$

$\Delta = 6 - 6i\sqrt{3} = 6 - 3\cdot 2(i\sqrt{3}) = 3^2 - 3\cdot 2(i\sqrt{3}) + (i\sqrt{3})^2$

$\Delta = (3 - i\sqrt{3})^2$ une racine carrée de Δ est $3 - i\sqrt{3}$

$z' = \dfrac{1+i\sqrt{3} - 3 + i\sqrt{3}}{4} = -\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}$

$z'' = \dfrac{1+i\sqrt{3} + 3 - i\sqrt{3}}{4} = 1$

$S_C = \left\{1;\ -\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}\right\}$

$z_1 = 1 \quad ; \quad z_2 = -\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2}$

3°/ a) $CA = CB \Leftrightarrow |z_1 - z| = |z_2 - z| \Leftrightarrow |1 - z| = \left|-\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2} - z\right|$

$\Leftrightarrow |-i\,tg\,\theta| = \left|-\dfrac{3}{2} + i\left(\dfrac{\sqrt{3}}{2} - tg\,\theta\right)\right| \Leftrightarrow tg^2\theta = \dfrac{9}{4} + \dfrac{3}{4} - \sqrt{3}tg\,\theta + tg^2\theta$

$\Leftrightarrow \quad tg\,\theta = \sqrt{3}$ or $\theta \in \left]0, \dfrac{\pi}{2}\right[\quad \Rightarrow \quad \theta = \dfrac{\pi}{3}$

Conclusion : Pour que $CA = CB$; il faut que $\theta = \dfrac{\pi}{3}$.